Algebra Cheat Sheet

Basic Properties & Facts

Arithmetic Operations

$ab+ac=a(b+c)$ $\qquad a\left(\frac{b}{c}\right)=\frac{ab}{c}$

$\frac{\left(\frac{a}{b}\right)}{c}=\frac{a}{bc}$ $\qquad \frac{a}{\left(\frac{b}{c}\right)}=\frac{ac}{b}$

$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$ $\qquad \frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$

$\frac{a-b}{c-d}=\frac{b-a}{d-c}$ $\qquad \frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$

$\frac{ab+ac}{a}=b+c,\ a\neq 0$ $\qquad \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}=\frac{ad}{bc}$

Exponent Properties

$a^n a^m=a^{n+m}$ $\qquad \frac{a^n}{a^m}=a^{n-m}=\frac{1}{a^{m-n}}$

$\left(a^n\right)^m=a^{nm}$ $\qquad a^0=1,\ a\neq 0$

$(ab)^n=a^n b^n$ $\qquad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$

$a^{-n}=\frac{1}{a^n}$ $\qquad \frac{1}{a^{-n}}=a^n$

$\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n=\frac{b^n}{a^n}$ $\qquad a^{\frac{n}{m}}=\left(a^{\frac{1}{m}}\right)^n=\left(a^n\right)^{\frac{1}{m}}$

Properties of Radicals

$\sqrt[n]{a}=a^{\frac{1}{n}}$ $\qquad \sqrt[n]{ab}=\sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[m]{\sqrt[n]{a}}=\sqrt[nm]{a}$ $\qquad \sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[n]{a^n}=a$, if n is odd

$\sqrt[n]{a^n}=|a|$, if n is even

Properties of Inequalities

If $a<b$ then $a+c<b+c$ and $a-c<b-c$

If $a<b$ and $c>0$ then $ac<bc$ and $\frac{a}{c}<\frac{b}{c}$

If $a<b$ and $c<0$ then $ac>bc$ and $\frac{a}{c}>\frac{b}{c}$

Properties of Absolute Value

$$|a|=\begin{cases} a & \text{if } a\geq 0 \\ -a & \text{if } a<0 \end{cases}$$

$|a|\geq 0$ $\qquad |-a|=|a|$

$|ab|=|a||b|$ $\qquad \left|\frac{a}{b}\right|=\frac{|a|}{|b|}$

$|a+b|\leq|a|+|b|$ Triangle Inequality

Distance Formula

If $P_1=(x_1,y_1)$ and $P_2=(x_2,y_2)$ are two points the distance between them is

$$d(P_1,P_2)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Complex Numbers

$i=\sqrt{-1}$ $\quad i^2=-1$ $\quad \sqrt{-a}=i\sqrt{a},\ a\geq 0$

$(a+bi)+(c+di)=a+c+(b+d)i$

$(a+bi)-(c+di)=a-c+(b-d)i$

$(a+bi)(c+di)=ac-bd+(ad+bc)i$

$(a+bi)(a-bi)=a^2+b^2$

$|a+bi|=\sqrt{a^2+b^2}$ Complex Modulus

$\overline{(a+bi)}=a-bi$ Complex Conjugate

$\overline{(a+bi)}(a+bi)=|a+bi|^2$

Logarithms and Log Properties

Definition

$y=\log_b x$ is equivalent to $x=b^y$

Example

$\log_5 125=3$ because $5^3=125$

Special Logarithms

$\ln x=\log_e x$ natural log

$\log x=\log_{10} x$ common log

where $e=2.718281828\ldots$

Logarithm Properties

$\log_b b=1$ $\qquad \log_b 1=0$

$\log_b b^x=x$ $\qquad b^{\log_b x}=x$

$\log_b\left(x^r\right)=r\log_b x$

$\log_b(xy)=\log_b x+\log_b y$

$\log_b\left(\frac{x}{y}\right)=\log_b x-\log_b y$

The domain of $\log_b x$ is $x>0$

Factoring and Solving

Factoring Formulas

$x^2-a^2=(x+a)(x-a)$

$x^2+2ax+a^2=(x+a)^2$

$x^2-2ax+a^2=(x-a)^2$

$x^2+(a+b)x+ab=(x+a)(x+b)$

$x^3+3ax^2+3a^2x+a^3=(x+a)^3$

$x^3-3ax^2+3a^2x-a^3=(x-a)^3$

$x^3+a^3=(x+a)\left(x^2-ax+a^2\right)$

$x^3-a^3=(x-a)\left(x^2+ax+a^2\right)$

$x^{2n}-a^{2n}=\left(x^n-a^n\right)\left(x^n+a^n\right)$

If n is odd then,

$x^n-a^n=(x-a)\left(x^{n-1}+ax^{n-2}+\cdots+a^{n-1}\right)$

x^n+a^n

$=(x+a)\left(x^{n-1}-ax^{n-2}+a^2x^{n-3}-\cdots+a^{n-1}\right)$

Quadratic Formula

Solve $ax^2+bx+c=0,\ a\neq 0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

If $b^2-4ac>0$ - Two real unequal solns.

If $b^2-4ac=0$ - Repeated real solution.

If $b^2-4ac<0$ - Two complex solutions.

Square Root Property

If $x^2=p$ then $x=\pm\sqrt{p}$

Absolute Value Equations/Inequalities

If b is a positive number

$|p|=b \quad \Rightarrow \quad p=-b$ or $p=b$

$|p|<b \quad \Rightarrow \quad -b<p<b$

$|p|>b \quad \Rightarrow \quad p<-b$ or $p>b$

Completing the Square

Solve $2x^2-6x-10=0$

(1) Divide by the coefficient of the x^2

$$x^2-3x-5=0$$

(2) Move the constant to the other side.

$$x^2-3x=5$$

(3) Take half the coefficient of x, square it and add it to both sides

$$x^2-3x+\left(-\frac{3}{2}\right)^2=5+\left(-\frac{3}{2}\right)^2=5+\frac{9}{4}=\frac{29}{4}$$

(4) Factor the left side

$$\left(x-\frac{3}{2}\right)^2=\frac{29}{4}$$

(5) Use Square Root Property

$$x-\frac{3}{2}=\pm\sqrt{\frac{29}{4}}=\pm\frac{\sqrt{29}}{2}$$

(6) Solve for x

$$x=\frac{3}{2}\pm\frac{\sqrt{29}}{2}$$

Functions and Graphs

Constant Function

$y=a$ or $f(x)=a$

Graph is a horizontal line passing through the point $(0,a)$.

Line/Linear Function

$y=mx+b$ or $f(x)=mx+b$

Graph is a line with point $(0,b)$ and slope m.

Slope

Slope of the line containing the two points (x_1,y_1) and (x_2,y_2) is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{\text{rise}}{\text{run}}$$

Slope – intercept form

The equation of the line with slope m and y-intercept $(0,b)$ is

$$y=mx+b$$

Point – Slope form

The equation of the line with slope m and passing through the point (x_1,y_1) is

$$y=y_1+m(x-x_1)$$

Parabola/Quadratic Function

$y=a(x-h)^2+k$ $\quad f(x)=a(x-h)^2+k$

The graph is a parabola that opens up if $a>0$ or down if $a<0$ and has a vertex at (h,k).

Parabola/Quadratic Function

$y=ax^2+bx+c$ $\quad f(x)=ax^2+bx+c$

The graph is a parabola that opens up if $a>0$ or down if $a<0$ and has a vertex at $\left(-\frac{b}{2a},f\left(-\frac{b}{2a}\right)\right)$.

Parabola/Quadratic Function

$x=ay^2+by+c$ $\quad g(y)=ay^2+by+c$

The graph is a parabola that opens right if $a>0$ or left if $a<0$ and has a vertex at $\left(g\left(-\frac{b}{2a}\right),-\frac{b}{2a}\right)$.

Circle

$(x-h)^2+(y-k)^2=r^2$

Graph is a circle with radius r and center (h,k).

Ellipse

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Graph is an ellipse with center (h,k) with vertices a units right/left from the center and vertices b units up/down from the center.

Hyperbola

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

Graph is a hyperbola that opens left and right, has a center at (h,k), vertices a units left/right of center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

Hyperbola

$$\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$$

Graph is a hyperbola that opens up and down, has a center at (h,k), vertices b units up/down from the center and asymptotes that pass through center with slope $\pm\frac{b}{a}$.

Common Algebraic Errors

Error	Reason/Correct/Justification/Example
$\frac{2}{0}\neq 0$ and $\frac{2}{0}\neq 2$	Division by zero is undefined!
$-3^2\neq 9$	$-3^2=-9$, $(-3)^2=9$ Watch parenthesis!
$(x^2)^3\neq x^5$	$(x^2)^3=x^2x^2x^2=x^6$
$\frac{a}{b+c}\neq\frac{a}{b}+\frac{a}{c}$	$\frac{1}{2}=\frac{1}{1+1}\neq\frac{1}{1}+\frac{1}{1}=2$
$\frac{1}{x^2+x^3}\neq x^{-2}+x^{-3}$	A more complex version of the previous error.
$\frac{\cancel{a}+bx}{\cancel{a}}\neq 1+bx$	$\frac{a+bx}{a}=\frac{a}{a}+\frac{bx}{a}=1+\frac{bx}{a}$ Beware of incorrect canceling!
$-a(x-1)\neq -ax-a$	$-a(x-1)=-ax+a$ Make sure you distribute the "-"!
$(x+a)^2\neq x^2+a^2$	$(x+a)^2=(x+a)(x+a)=x^2+2ax+a^2$
$\sqrt{x^2+a^2}\neq x+a$	$5=\sqrt{25}=\sqrt{3^2+4^2}\neq\sqrt{3^2}+\sqrt{4^2}=3+4=7$
$\sqrt{x+a}\neq\sqrt{x}+\sqrt{a}$	See previous error.
$(x+a)^n\neq x^n+a^n$ and $\sqrt[n]{x+a}\neq\sqrt[n]{x}+\sqrt[n]{a}$	More general versions of previous three errors.
$2(x+1)^2\neq(2x+2)^2$	$2(x+1)^2=2(x^2+2x+1)=2x^2+4x+2$ $(2x+2)^2=4x^2+8x+4$ Square first then distribute!
$(2x+2)^2\neq 2(x+1)^2$	See the previous example. You can not factor out a constant if there is a power on the parethesis!
$\sqrt{-x^2+a^2}\neq-\sqrt{x^2+a^2}$	$\sqrt{-x^2+a^2}=(-x^2+a^2)^{\frac{1}{2}}$ Now see the previous error.
$\frac{a}{\left(\frac{b}{c}\right)}\neq\frac{ab}{c}$	$\frac{a}{\left(\frac{b}{c}\right)}=\frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)}=\left(\frac{a}{1}\right)\left(\frac{c}{b}\right)=\frac{ac}{b}$
$\frac{\left(\frac{a}{b}\right)}{c}\neq\frac{ac}{b}$	$\frac{\left(\frac{a}{b}\right)}{c}=\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)}=\left(\frac{a}{b}\right)\left(\frac{1}{c}\right)=\frac{a}{bc}$

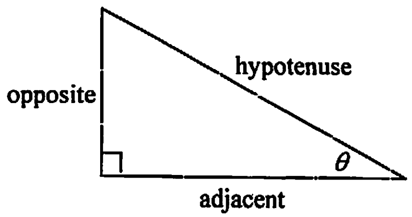
Trig Cheat Sheet

Definition of the Trig Functions

Right triangle definition

For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.



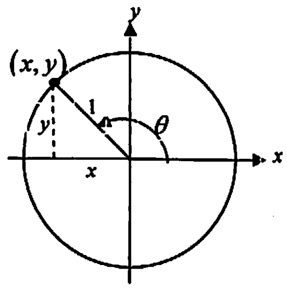
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Unit circle definition

For this definition θ is any angle.



$$\sin\theta = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, θ can be any angle

$\cos\theta$, θ can be any angle

$\tan\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \ldots$

$\csc\theta$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

$\sec\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \ldots$

$\cot\theta$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

Range

The range is all possible values to get out of the function.

$-1 \leq \sin\theta \leq 1$ $\qquad$ $\csc\theta \geq 1$ and $\csc\theta \leq -1$

$-1 \leq \cos\theta \leq 1$ $\qquad$ $\sec\theta \geq 1$ and $\sec\theta \leq -1$

$-\infty \leq \tan\theta \leq \infty$ $\qquad$ $-\infty \leq \cot\theta \leq \infty$

Period

The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

$$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

Formulas and Identities

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Periodic Formulas

If n is an integer.

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$

$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas

$$\sin^2\theta = \frac{1}{2}\left(1 - \cos(2\theta)\right)$$

$$\cos^2\theta = \frac{1}{2}\left(1 + \cos(2\theta)\right)$$

$$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha + \beta) - \sin(\alpha - \beta)\right]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

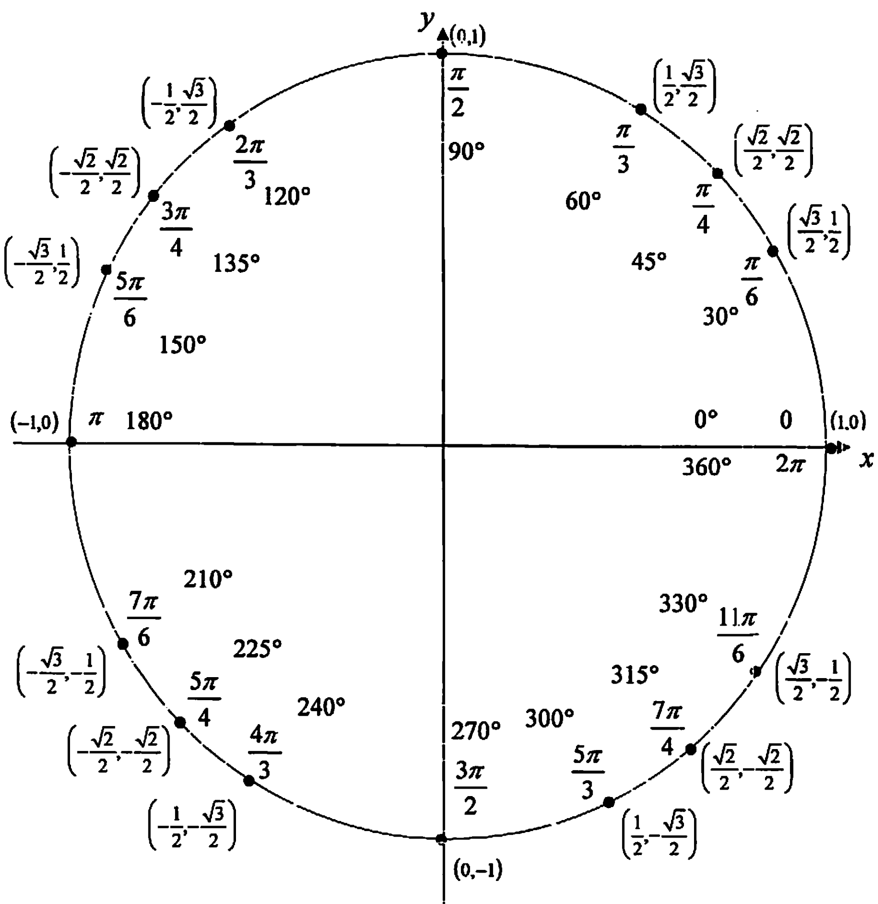
Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Unit Circle



For any ordered pair on the unit circle (x,y) : $\cos\theta = x$ and $\sin\theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right)=\frac{1}{2} \qquad \sin\left(\frac{5\pi}{3}\right)=-\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

$y=\sin^{-1}x$ is equivalent to $x=\sin y$

$y=\cos^{-1}x$ is equivalent to $x=\cos y$

$y=\tan^{-1}x$ is equivalent to $x=\tan y$

Domain and Range

Function	Domain	Range
$y=\sin^{-1}x$	$-1\le x\le 1$	$-\frac{\pi}{2}\le y\le\frac{\pi}{2}$
$y=\cos^{-1}x$	$-1\le x\le 1$	$0\le y\le\pi$
$y=\tan^{-1}x$	$-\infty< x<\infty$	$-\frac{\pi}{2}< y<\frac{\pi}{2}$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right)=x \qquad \cos^{-1}\left(\cos(\theta)\right)=\theta$

$\sin\left(\sin^{-1}(x)\right)=x \qquad \sin^{-1}\left(\sin(\theta)\right)=\theta$

$\tan\left(\tan^{-1}(x)\right)=x \qquad \tan^{-1}\left(\tan(\theta)\right)=\theta$

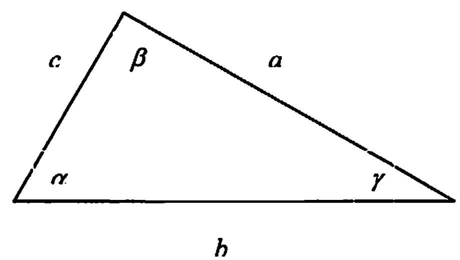
Alternate Notation

$\sin^{-1}x=\arcsin x$

$\cos^{-1}x=\arccos x$

$\tan^{-1}x=\arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$$

Law of Cosines

$$a^2=b^2+c^2-2bc\cos\alpha$$

$$b^2=a^2+c^2-2ac\cos\beta$$

$$c^2=a^2+b^2-2ab\cos\gamma$$

Mollweide's Formula

$$\frac{a+b}{c}=\frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$$

Law of Tangents

$$\frac{a-b}{a+b}=\frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$$

$$\frac{b-c}{b+c}=\frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$

$$\frac{a-c}{a+c}=\frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$

Derivatives

Basic Properties/Formulas/Rules

$\frac{d}{dx}\left(cf(x)\right)=cf'(x)$, c is any constant. $\left(f(x)\pm g(x)\right)'=f'(x)\pm g'(x)$

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$, n is any number. $\frac{d}{dx}(c)=0$, c is any constant.

$(fg)'=f'g+fg'$ – **(Product Rule)** $\left(\frac{f}{g}\right)'=\frac{f'g-fg'}{g^2}$ – **(Quotient Rule)**

$\frac{d}{dx}\left(f\left(g(x)\right)\right)=f'\left(g(x)\right)g'(x)$ **(Chain Rule)**

$\frac{d}{dx}\left(\mathrm{e}^{g(x)}\right)=g'(x)\mathrm{e}^{g(x)}$ $\frac{d}{dx}\left(\ln g(x)\right)=\frac{g'(x)}{g(x)}$

Common Derivatives

Polynomials

$\frac{d}{dx}(c)=0$ $\frac{d}{dx}(x)=1$ $\frac{d}{dx}(cx)=c$ $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$ $\frac{d}{dx}\left(cx^n\right)=ncx^{n-1}$

Trig Functions

$\frac{d}{dx}(\sin x)=\cos x$ $\frac{d}{dx}(\cos x)=-\sin x$ $\frac{d}{dx}(\tan x)=\sec^2 x$

$\frac{d}{dx}(\sec x)=\sec x\tan x$ $\frac{d}{dx}(\csc x)=-\csc x\cot x$ $\frac{d}{dx}(\cot x)=-\csc^2 x$

Inverse Trig Functions

$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\left(\cos^{-1}x\right)=-\frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$

$\frac{d}{dx}\left(\sec^{-1}x\right)=\frac{1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}\left(\csc^{-1}x\right)=-\frac{1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}\left(\cot^{-1}x\right)=-\frac{1}{1+x^2}$

Exponential/Logarithm Functions

$\frac{d}{dx}\left(a^x\right)=a^x\ln(a)$ $\frac{d}{dx}\left(\mathrm{e}^x\right)=\mathrm{e}^x$

$\frac{d}{dx}\left(\ln(x)\right)=\frac{1}{x},\ x>0$ $\frac{d}{dx}\left(\ln|x|\right)=\frac{1}{x},\ x\neq 0$ $\frac{d}{dx}\left(\log_a(x)\right)=\frac{1}{x\ln a},\ x>0$

Hyperbolic Trig Functions

$\frac{d}{dx}(\sinh x)=\cosh x$ $\frac{d}{dx}(\cosh x)=\sinh x$ $\frac{d}{dx}(\tanh x)=\operatorname{sech}^2 x$

$\frac{d}{dx}(\operatorname{sech} x)=-\operatorname{sech} x\tanh x$ $\frac{d}{dx}(\operatorname{csch} x)=-\operatorname{csch} x\coth x$ $\frac{d}{dx}(\coth x)=-\operatorname{csch}^2 x$

Common Derivatives and Integrals

Integrals

Basic Properties/Formulas/Rules

$\int cf(x)\,dx=c\int f(x)\,dx$, c is a constant. $\int f(x)\pm g(x)\,dx=\int f(x)\,dx\pm\int g(x)\,dx$

$\int_a^b f(x)\,dx=F(x)\Big|_a^b=F(b)-F(a)$ where $F(x)=\int f(x)\,dx$

$\int_a^b cf(x)\,dx=c\int_a^b f(x)\,dx$, c is a constant. $\int_a^b f(x)\pm g(x)\,dx=\int_a^b f(x)\,dx\pm\int_a^b g(x)\,dx$

$\int_a^a f(x)\,dx=0$ $\int_a^b f(x)\,dx=-\int_b^a f(x)\,dx$

$\int_a^b f(x)\,dx=\int_a^c f(x)\,dx+\int_c^b f(x)\,dx$ $\int_a^b c\,dx=c(b-a)$

If $f(x)\geq 0$ on $a\leq x\leq b$ then $\int_a^b f(x)\,dx\geq 0$

If $f(x)\geq g(x)$ on $a\leq x\leq b$ then $\int_a^b f(x)\,dx\geq\int_a^b g(x)\,dx$

Common Integrals

Polynomials

$\int dx=x+c$ $\int k\,dx=kx+c$ $\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$

$\int\frac{1}{x}\,dx=\ln|x|+c$ $\int x^{-1}\,dx=\ln|x|+c$ $\int x^{-n}dx=\frac{1}{-n+1}x^{-n+1}+c,\ n\neq 1$

$\int\frac{1}{ax+b}\,dx=\frac{1}{a}\ln|ax+b|+c$ $\int x^{\frac{p}{q}}dx=\frac{1}{\frac{p}{q}+1}x^{\frac{p}{q}+1}+c=\frac{q}{p+q}x^{\frac{p+q}{q}}+c$

Trig Functions

$\int\cos u\,du=\sin u+c$ $\int\sin u\,du=-\cos u+c$ $\int\sec^2 u\,du=\tan u+c$

$\int\sec u\tan u\,du=\sec u+c$ $\int\csc u\cot u\,du=-\csc u+c$ $\int\csc^2 u\,du=-\cot u+c$

$\int\tan u\,du=\ln|\sec u|+c$ $\int\cot u\,du=\ln|\sin u|+c$

$\int\sec u\,du=\ln|\sec u+\tan u|+c$ $\int\sec^3 u\,du=\frac{1}{2}\left(\sec u\tan u+\ln|\sec u+\tan u|\right)+c$

$\int\csc u\,du=\ln|\csc u-\cot u|+c$ $\int\csc^3 u\,du=\frac{1}{2}\left(-\csc u\cot u+\ln|\csc u-\cot u|\right)+c$

Exponential/Logarithm Functions

$\int\mathrm{e}^u\,du=\mathrm{e}^u+c$ $\int a^u\,du=\frac{a^u}{\ln a}+c$ $\int\ln u\,du=u\ln(u)-u+c$

$\int\mathrm{e}^{au}\sin(bu)\,du=\frac{\mathrm{e}^{au}}{a^2+b^2}\left(a\sin(bu)-b\cos(bu)\right)+c$ $\int u\mathrm{e}^u\,du=(u-1)\mathrm{e}^u+c$

$\int\mathrm{e}^{au}\cos(bu)\,du=\frac{\mathrm{e}^{au}}{a^2+b^2}\left(a\cos(bu)+b\sin(bu)\right)+c$ $\int\frac{1}{u\ln u}\,du=\ln|\ln u|+c$

Inverse Trig Functions

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right)+c \qquad \int \sin^{-1}u\,du = u\sin^{-1}u+\sqrt{1-u^2}+c$$

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+c \qquad \int \tan^{-1}u\,du = u\tan^{-1}u-\frac{1}{2}\ln\left(1+u^2\right)+c$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right)+c \qquad \int \cos^{-1}u\,du = u\cos^{-1}u-\sqrt{1-u^2}+c$$

Hyperbolic Trig Functions

$$\int \sinh u\,du = \cosh u+c \qquad \int \cosh u\,du = \sinh u+c \qquad \int \operatorname{sech}^2 u\,du = \tanh u+c$$

$$\int \operatorname{sech}\tanh u\,du = -\operatorname{sech} u+c \qquad \int \operatorname{csch}\coth u\,du = -\operatorname{csch} u+c \qquad \int \operatorname{csch}^2 u\,du = -\coth u+c$$

$$\int \tanh u\,du = \ln\left(\cosh u\right)+c \qquad \int \operatorname{sech} u\,du = \tan^{-1}\left|\sinh u\right|+c$$

Miscellaneous

$$\int \frac{1}{a^2-u^2}\,du = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right|+c \qquad \int \frac{1}{u^2-a^2}\,du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right|+c$$

$$\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2}+\frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right|+c$$

$$\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2}-\frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right|+c$$

$$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right)+c$$

$$\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2}+\frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right)+c$$

Standard Integration Techniques

Note that all but the first one of these tend to be taught in a Calculus II class.

u Substitution

Given $\int_a^b f\left(g(x)\right)g'(x)\,dx$ then the substitution $u=g(x)$ will convert this into the integral, $\int_a^b f\left(g(x)\right)g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$.

Integration by Parts

The standard formulas for integration by parts are,

$$\int u\,dv = uv-\int v\,du \qquad \int_a^b u\,dv = uv\Big|_a^b-\int_a^b v\,du$$

Choose u and dv and then compute du by differentiating u and compute v by using the fact that $v=\int dv$.

Common Derivatives and Integrals

Trig Substitutions

If the integral contains the following root use the given substitution and formula.

$$\sqrt{a^2-b^2x^2} \quad\Rightarrow\quad x=\frac{a}{b}\sin\theta \quad\text{and}\quad \cos^2\theta = 1-\sin^2\theta$$

$$\sqrt{b^2x^2-a^2} \quad\Rightarrow\quad x=\frac{a}{b}\sec\theta \quad\text{and}\quad \tan^2\theta = \sec^2\theta-1$$

$$\sqrt{a^2+b^2x^2} \quad\Rightarrow\quad x=\frac{a}{b}\tan\theta \quad\text{and}\quad \sec^2\theta = 1+\tan^2\theta$$

Partial Fractions

If integrating $\int \frac{P(x)}{Q(x)}\,dx$ where the degree (largest exponent) of $P(x)$ is smaller than the degree of $Q(x)$ then factor the denominator as completely as possible and find the partial fraction decomposition of the rational expression. Integrate the partial fraction decomposition (P.F.D.). For each factor in the denominator we get term(s) in the decomposition according to the following table.

Factor in $Q(x)$	Term in P.F.D	Factor in $Q(x)$	Term in P.F.D
$ax+b$	$\frac{A}{ax+b}$	$(ax+b)^k$	$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+\cdots+\frac{A_k}{(ax+b)^k}$
ax^2+bx+c	$\frac{Ax+B}{ax^2+bx+c}$	$\left(ax^2+bx+c\right)^k$	$\frac{A_1x+B_1}{ax^2+bx+c}+\cdots+\frac{A_kx+B_k}{\left(ax^2+bx+c\right)^k}$

Products and (some) Quotients of Trig Functions

$\int \sin^n x\cos^m x\,dx$

1. **If n is odd.** Strip one sine out and convert the remaining sines to cosines using $\sin^2 x = 1-\cos^2 x$, then use the substitution $u=\cos x$
2. **If m is odd.** Strip one cosine out and convert the remaining cosines to sines using $\cos^2 x = 1-\sin^2 x$, then use the substitution $u=\sin x$
3. **If n and m are both odd.** Use either 1. or 2.
4. **If n and m are both even.** Use double angle formula for sine and/or half angle formulas to reduce the integral into a form that can be integrated.

$\int \tan^n x\sec^m x\,dx$

1. **If n is odd.** Strip one tangent and one secant out and convert the remaining tangents to secants using $\tan^2 x = \sec^2 x-1$, then use the substitution $u=\sec x$
2. **If m is even.** Strip two secants out and convert the remaining secants to tangents using $\sec^2 x = 1+\tan^2 x$, then use the substitution $u=\tan x$
3. **If n is odd and m is even.** Use either 1. or 2.
4. **If n is even and m is odd.** Each integral will be dealt with differently.

Convert Example : $\cos^6 x = \left(\cos^2 x\right)^3 = \left(1-\sin^2 x\right)^3$

